Miscellaneous Examples

Example 20 How many words, with or without meaning, each of 3 vowels and 2 consonants can be formed from the letters of the word INVOLUTE ?

Solution In the word INVOLUTE, there are 4 vowels, namely, I,O,E,Uand 4 consonants, namely, N, V, L and T.

The number of ways of selecting 3 vowels out of 4 = ${}^4C_3 = 4$.

The number of ways of selecting 2 consonants out of 4 = ${}^4C_2 = 6$.

Therefore, the number of combinations of 3 vowels and 2 consonants is $4 \times 6 = 24$.

Now, each of these 24 combinations has 5 letters which can be arranged among themselves in 5 ! ways. Therefore, the required number of different words is $24 \times 5! = 2880$.

Example 21 A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has (i) no girl ? (ii) at least one boy and one girl ? (iii) at least 3 girls ?

Solution (i) Since, the team will not include any girl, therefore, only boys are to be selected. 5 boys out of 7 boys can be selected in ${}^7C_5$ ways. Therefore, the required

number of ways $= {}^7C_5 = \frac{7!}{5!\;2!} = \frac{6 \times 7}{2} = 21$

(ii) Since, at least one boy and one girl are to be there in every team. Therefore, the team can consist of

(a) 1 boy and 4 girls (b) 2 boys and 3 girls

(c) 3 boys and 2 girls (d) 4 boys and 1 girl.

1 boy and 4 girls can be selected in ${}^7C_1 \times {}^4C_4$ ways.

2 boys and 3 girls can be selected in ${}^7C_2 \times {}^4C_3$ ways.

3 boys and 2 girls can be selected in ${}^7C_3 \times {}^4C_2$ ways.

4 boys and 1 girl can be selected in ${}^7C_4 \times {}^4C_1$ ways.

Therefore, the required number of ways

$= {}^7C_1 \times {}^4C_4 + {}^7C_2 \times {}^4C_3 + {}^7C_3 \times {}^4C_2 + {}^7C_4 \times {}^4C_1$

$= 7 + 84 + 210 + 140 = 441$

(iii) Since, the team has to consist of at least 3 girls, the team can consist of

(a) 3 girls and 2 boys, or (b) 4 girls and 1 boy.

Example 22 Find the number of words with or without meaning which can be made using all the letters of the word AGAIN. If these words are written as in a dictionary, what will be the 50^{th} word?

Solution There are 5 letters in the word AGAIN, in which A appears 2 times. Therefore, the required number of words $= \frac{5!}{2!} = 60$.

To get the number of words starting with A, we fix the letter A at the extreme left position, we then rearrange the remaining 4 letters taken all at a time. There will be as many arrangements of these 4 letters taken 4 at a time as there are permutations of 4 different things taken 4 at a time. Hence, the number of words starting with A = 4! = 24. Then, starting with G, the number of words $= \frac{4!}{2!} = 12$ as after placing G at the extreme left position, we are left with the letters A, A, I and N. Similarly, there are 12 words starting with the next letter I. Total number of words so far obtained = 24 + 12 + 12 =48.

The 49^{th} word is NAAGI. The 50^{th} word is NAAIG.

Example 23 How many numbers greater than 1000000 can be formed by using the digits 1, 2, 0, 2, 4, 2, 4?

Solution Since, 1000000 is a 7-digit number and the number of digits to be used is also 7. Therefore, the numbers to be counted will be 7-digit only. Also, the numbers have to be greater than 1000000, so they can begin either with 1, 2 or 4.

The number of numbers beginning with 1 $= \frac{6!}{3!\,2!} = \frac{4\times5\times6}{2} = 60$, as when 1 is fixed at the extreme left position, the remaining digits to be rearranged will be 0, 2, 2, 2, 4, 4, in which there are 3, 2*s* and 2, 4*s*.

Total numbers begining with 2

$$= \frac{6!}{2!\;2!} = \frac{3\times4\times5\times6}{2} = 180$$

and total numbers begining with 4 $= \frac{6!}{3!} = 4\times5\times6 = 120$

Therefore, the required number of numbers = 60 + 180 + 120 = 360.

Alternative Method

The number of 7-digit arrangements, clearly, $\frac{7!}{3!\ 2!} = 420$. But, this will include those numbers also, which have 0 at the extreme left position. The number of such arrangements $\frac{6!}{3!\ 2!}$ (by fixing 0 at the extreme left position) = 60.

Therefore, the required number of numbers = 420 – 60 = 360.

Note If one or more than one digits given in the list is repeated, it will be understood that in any number, the digits can be used as many times as is given in the list, e.g., in the above example 1 and 0 can be used only once whereas 2 and 4 can be used 3 times and 2 times, respectively.

Example 24 In how many ways can 5 girls and 3 boys be seated in a row so that no two boys are together?

Solution Let us first seat the 5 girls. This can be done in 5! ways. For each such arrangement, the three boys can be seated only at the cross marked places.

$$\times\ G \times G \times G \times G \times G \times.$$

There are 6 cross marked places and the three boys can be seated in ${}^6P_3$ ways. Hence, by multiplication principle, the total number of ways

$$= 5! \times {}^6P_3 = 5! \times \frac{6!}{3!}$$

$$= 4 \times 5 \times 2 \times 3 \times 4 \times 5 \times 6 = 14400.$$

Miscellaneous Exercise on Chapter 7

Summary

- *Fundamental principle of counting* If an event can occur in m different ways, following which another event can occur in n different ways, then the total number of occurrence of the events in the given order is $m \times n$.
- The number of permutations of n different things taken r at a time, where repetition is not allowed, is denoted by ${}^nP_r$ and is given by ${}^nP_r = \dfrac{n!}{(n-r)!}$, where $0 \leq r \leq n$.
- $n! = 1 \times 2 \times 3 \times \ldots \times n$
- $n! = n \times (n - 1)!$
- The number of permutations of n different things, taken r at a time, where repeatition is allowed, is n^r.
- The number of permutations of n objects taken all at a time, where p_1 objects

are of first kind, p_2 objects are of the second kind, ..., p_k objects are of the k^{th} kind and rest, if any, are all different is $\frac{n!}{p_1!\, p_2! \ldots p_k!}$.

- The number of combinations of n different things taken r at a time, denoted by ^nC_r, is given by $^nC_r = \frac{n!}{r!(n-r)!}$, $0 \leq r \leq n$.